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EPIDEMICS OF CORRUPTION USING INCIDENCE FUNCTION

Abstract. Corruption is a slow poison damaging individuals and consequently society and nation. This is a global issue. Any individual can be exposed to the corruption. Slowly and steadily, an individual turns out to be most corrupted. In this study, an attempt is made to formulate the dynamics of corruption. The non-linear system of differential equations constructed for different compartments viz. Susceptible, Exposed, Infected and Punished. The punishment in terms of transfers from one place to another is defined in exposed and infected classes. These transfers lead the vertical transmission in both the classes. The threshold in terms of reproduction number is computed to make society corruption free. The local and global stability is analyzed. The proposed model is supported by numerical simulation. The observations and suggestions are outlined to have corruption free society or reduced corruption.

Keywords: Corruption, Vertical Transmission, Incidence function, Basic reproduction number, Local Stability, Global Stability.

JEL Classification: A14, D73

1. Introduction

Corruption exists in society from early stages of human development. It will require centuries to change the nations' perception of doing things as corruption is deeply rooted in society. Nowadays it becomes trend, called the shortcut doing things instead of legally and right. International laws the legislations condemn these "shortcut ways" as it may be right for one, not for the other. Nita H. Shah, Bijal M. Yeolekar, Zalak A. Patel

Corruption can be considered as one of the main obstacle in the development of a powerful government system. Rose-Ackerman (1999) consider corruption as a "symptom that something has gone wrong in the management of the state". Jain (2001) described three preconditions for existence of corruption, which are discretionary power related to procedures, economic issues linked to power, and sufficiently low punishment. Corruption is psychic nonconformity towards ideal. It contains events like bribery and embezzlement. Mauro (1997) observed corruption as a universal problem rooted in countries and nations in various size and shapes. Michael Johnston (2008) observed that there are several types of corruption: Planned Corruption, Irregular (individual) corruption, Political corruption, Grand corruption, Petty Corruption. Bird, et. al. (2006) observed that stability in the growth of country can be obtained by balancing political forces and institutions. Bin et.al. (2015) observed that, the past level of corruption has a strong influence on the current corruption level. They also derived that conditional corruption matters. CaliskanandKadiu(2014) observed that the factors affecting corruption index are population growth, HDI, and governmental spending. Blackburnet.al.(2006) discussed harm-ness of corruption on economic development and reversely how low-levels of development can promote high corruption. Lui (1986) studied that how spokespersons maximize their expected payoff by adopting mal-practice. Dynamic aspect of young officials' expected payoff of bribes they might receive when they are old. Mishra (2006) studied how corruption can develop via an evolutionary game.

In this paper, the non-linear corruption transmission model is formulation and basic reproduction number is worked out in section 2. In section 3, the sensitivity of the model is established. The numerical simulation is discussed in section 4. Paper ends up with conclusion.

2. Mathematical Modeling

The notations of the Corruption transmission model are as follows.

2.1 Notations and parametric values

Notation			Parametric value
N(t)	:	Total population at time <i>t</i>	10,000
S(t)	:	The class of Susceptible to become corrupted	10
E(t)	:	The class of Exposed individuals who are less corrupted	8

Table1. Notations of the model with parameter values

I(t)	:	The class of Infected individuals who are most corrupted	3
P(t)	:	The portion of individuals who gets punishment due to corruption	4
λ	:	Force of contacts of corrupted persons with Susceptible	
C_1		Number of contacts made by a person from Exposed	3
eta_i	:	The rate of effective contacts responsible for corruption spreading from E to S	0.1
C_2	:	Number of contacts made by a person from Infectious	1
β_2	:	Contact rate of Susceptible with Infectious	0.005
<i>C</i> ₃	:	Number of contacts made by a person from Punished	3
β_3	:	Contact rate of Susceptible with Punished	0.2
В	:	New recruitment in Susceptible class	10
μ	:	The rate of corruption free individuals	0.1
b	:	The rate of new sprouted corrupted individuals	0.05
θ	:	The rate at which individual become corruption free	0.8
α	:	The rate at which corrupted individuals moves from exposed class to infectious class	0.3
ε	:	The rate at which exposed individual gets punished	0.4
η	:	The rate at which most corrupted gets punished	0.1
r_1	:	Realization rate (after punishment) individual enters into exposed class	0.2
<i>r</i> ₂	:	Realization rate at which individual gets punished and they becomes non-corrupted	0.75
m_i	:	The rate of migrated individual in	$m_1 = 0.1$ and
		compartment (due to punishment)	$m_{\rm r} = 0.05$
1			$m_2 = 0.00$

2.2 Mathematical Modelling

To study the corruption transmission with punishment effect, the nonlinear mathematical model is formulated. The total population is divided in four compartments viz. Susceptible class S(t), Exposed (less corrupted) class E(t), Infected (most corrupted) class I(t), Punished class P(t). The dynamics of the corruption transmission model is derived in following Figure 1.



Figure 1. Corruption transmission diagram

From Figure1, the non-linear system of differential equations of corruption model as

$$\frac{dS}{dt} = B + \theta b \left(E + r_1 P \right) - \left(\lambda + \mu \right) S$$
$$\frac{dE}{dt} = \lambda S + b \theta r_2 P + (1 - \theta) b E + m_1 E - \left(\varepsilon + \alpha + \mu \right) E \tag{1}$$

$$\frac{dI}{dt} = \alpha E + b(1 - \theta r_1 - \theta r_2)P + m_2 I - (\eta + \mu)I$$
$$\frac{dP}{dt} = \varepsilon E + \eta I - (b + \mu)P$$

where,
$$\lambda = \frac{C_1 \beta_1 SE + C_2 \beta_2 SI + C_3 \beta_3 SP}{N}$$

Adding all equations of system (1), we have

$$\frac{d}{dt}(S+E+I+P) = B - \mu(S+E+I+P) + b(E) + m_1E + m_2I + m_2I$$

So, $\lim_{t\to\infty} \sup(S+E+I+P) \le \frac{B}{\mu}$.

Therefore, the feasible solution for state variables in the corruption system is

$$\Omega = \{ (S + E + I + P) / S + E + I + P \le \frac{B}{\mu}, S > 0, E \ge 0, I > 0, P > 0 \}$$

The set Ω is non-negative invariant for any time t > 0. The system has the corruption free equilibrium point as $X_0 = \left(\frac{B}{\mu}, 0, 0, 0\right)$.

To discuss the epidemiology of the transmission of corruption modelling different compartment at the equilibrium point X_0 is discussed below.

2.2.1 Migration and EI-model

To see the effect of migrated individual, we will analyze the following differential equations and calculate the basic reproduction number R_0 by the next generation matrix.

$$\frac{dS}{dt} = B + \theta b \left(E + r_1 P \right) - \left(\lambda + \mu \right) S$$

$$\frac{dE}{dt} = \lambda S + b \theta r_2 P + \left(1 - \theta \right) b E + m_1 E - \left(\varepsilon + \alpha + \mu \right) E \qquad (2)$$

$$\frac{dI}{dt} = \alpha E + b \left(1 - \theta r_1 - \theta r_2 \right) P + m_2 I - \left(\eta + \mu \right) I$$

Let X' = (E, I)', here dash defines derivative.

$$X' = \frac{dX}{dt} = \Im(X) - \upsilon(X)$$

where $\Im(X)$, the appearance rate of new corrupted individuals in different class and $\upsilon(X)$ represents the rate of corruption transmission in the system, which is given by

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Clearly, V is non-singular matrix. Hence, the basic reproduction number R_0 is the spectral radius of matrix FV^{-1} .

$$R_{0M} = \frac{c_1 \beta_1 B (\eta + \mu - m_2) + c_2 \beta_2 B \alpha}{\mu N (\eta + \mu - m_1) [(\alpha + \mu + \varepsilon - m_1) - (1 - \theta)]} = 0.02246$$
(3)

The basic reproduction number gives the average number of new recruitments of the most corrupted individuals in total population consisting of susceptible only. It suggests the relevant control strategies to be opted. If more migrated individuals enter the system then the society becomes more corrupted.

2.2.1.1 Stability of the equilibria

In this section, we discuss local and global stability of the corruption free system.

2.2.1.1.1 Local Stability

The corruption-free equilibrium is locally asymptotically stable whenever $R_{0M} < 1$. The linearization approach to prove the local stability of the corruption-free equilibrium at $X_0 = \left(\frac{B}{\mu}, 0, 0\right)$ is considered. The Jacobian matrix associated with the system (2) is

$$J = \begin{bmatrix} -\mu & \theta b - \frac{C_1 \beta_1 B}{\mu N} & \frac{C_2 \beta_2 B}{\mu N} \\ 0 & \frac{C_1 \beta_1 B}{\mu N} - \left(\varepsilon + \alpha + \mu - m_1 - (1 - \theta)b\right) & \frac{C_2 \beta_2 B}{\mu N} \\ 0 & \alpha & -(\eta + \mu - m_2) \end{bmatrix}$$

Here, we have μ as one of the eigenvalue. For the remaining sub-matrix,

Trace
$$(J) = \frac{C_1 \beta_1 B}{\mu N} + (1 - \theta) b - (\varepsilon + \alpha + \mu - m_1) - (\eta + \mu - m_2) < 0$$
. This gives,

$$\frac{C_1\beta_1B}{\mu N} + (1-\theta)b < (\varepsilon + \alpha + \mu - m_1) + (\eta + \mu - m_2)$$
$$= \frac{C_1\beta_1B + (1-\theta)b}{\mu N[(\varepsilon + \alpha + \mu - m_1) + (\eta + \mu - m_2)]} < 1$$

and

$$\det(J) = \left[\frac{C_1\beta_1B}{\mu N} - \left(\varepsilon + \alpha + \mu - m_1 - (1-\theta)b\right)\right] \left[-\left(\eta + \mu - m_2\right)\right] - \alpha \frac{C_2\beta_2B}{\mu N}$$
$$= \frac{c_1\beta_1B(\eta + \mu - m_2) + c_2\beta_2B\alpha}{\mu N(\eta + \mu - m_1)\left[\left(\alpha + \mu + \varepsilon - m_1\right) - (1-\theta)\right]} < \text{lor } \mathbf{R}_{0M} < 1.$$

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2.2.1.1.2 Global Stability

The corruption-free equilibrium is globally stable if $det(I - (FV^{-1})) > 0$

i.e.

$$\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{c_1 \beta_1 S(\eta + \mu - m_2) + c_2 \beta_2 S \alpha}{\left[\left(\alpha + \mu + \varepsilon - m_1 \right) - \left(1 - \theta \right) \right] \left(\eta + \mu - m_2 \right)} & \frac{c_2 \beta_2 S \left[\left(\alpha + \mu + \varepsilon - m_1 \right) - \left(1 - \theta \right) \right]}{\left[\left(\alpha + \mu + \varepsilon - m_1 \right) - \left(1 - \theta \right) \right] \left(\eta + \mu - m_2 \right)} \end{bmatrix} > 0$$

$$\Rightarrow 1 - \frac{c_1 \beta_1 S(\eta + \mu - m_2) + c_2 \beta_2 S \alpha}{\left[\left(\alpha + \mu + \varepsilon - m_1 \right) - \left(1 - \theta \right) \right] \left(\eta + \mu - m_2 \right)} > 0 \text{ gives } R_{0M} < 1.$$

2.2.2 Model for punishment to corrupted individual

The punishment is to given to the individuals in E and I both. So, the model

with punishment class is

$$\frac{dE}{dt} = \lambda S + b\theta r_2 P + (1 - \theta)bE + m_1 E - (\varepsilon + \alpha + \mu)E$$
$$\frac{dI}{dt} = \alpha E + b(1 - \theta r_1 - \theta r_2)P + m_2 I - (\eta + \mu)I$$
(4)
$$\frac{dP}{dt} = \varepsilon E + \eta I - (b + \mu)P$$

Here, we calculate the basic reproduction number of the system (4) as discussed in section 2.2.1.

Let
$$X' = \Im(X) - \upsilon(X)$$

where,

As mentioned above in section 2.2.1,

$$F = \begin{bmatrix} \frac{c_1 \beta_1 \beta}{\mu N} & \frac{c_2 \beta_2 \beta}{\mu N} & \frac{c_3 \beta_3 \beta}{\mu N} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and }$$
$$V = \begin{bmatrix} (\alpha + \mu + \varepsilon - m_1) - (1 - \theta) & 0 & -b\theta r_2 \\ -\alpha & (\eta + \mu - m_2) & -b(1 - \theta r_1 - \theta r_2) \\ -\varepsilon & -\eta & (b + \mu) \end{bmatrix}.$$

The basic reproduction number of system (4) is

$$R_{0P} = \frac{C_1 \beta_1 B(\mu A_2 + A_1 \eta) \theta b r_2 - C_2 \beta_2 B(\varepsilon A_1 - \alpha (b + \mu))}{\mu N(AA_2 \mu + AA_1 \eta - \theta b r_2 (\alpha \eta + \varepsilon A_2))} = 0.31764$$
(5)

where, $A = (\alpha + \mu + \varepsilon - m_1) - (1 - \theta)b A_1 = b(1 - \theta r_1 - \theta r_2)$ and $A_2 = (\eta + \mu - m_2).$

Here, we note that if we increase punishment level more in the system it automatic reduces the corrupted individual in the society.

2.2.2.1 Stability of the equilibria

2.2.2.1.1 Local Stability

The Jacobian matrix for the corruption–free equilibrium at $X_0 = \left(\frac{B}{\mu}, 0, 0\right)$

of system (4) is

$$J = \begin{bmatrix} \frac{C_1 \beta_1 B}{\mu N} - A & \frac{C_2 \beta_2 B}{\mu N} & \frac{C_3 \beta_3 B}{\mu N} + b \theta r_2 \end{bmatrix}$$
$$\begin{pmatrix} \alpha & -A_2 & A_1 \\ \varepsilon & \eta & -(b+\mu) \end{pmatrix}$$

Here, we have μ as one of the eigenvalue. For the remaining sub-matrix,

Trace
$$(J) = \frac{C_1 \beta_1 B}{\mu N} + (1 - \theta) b - (\varepsilon + \alpha + \mu - m_1) - (\eta + \mu - m_2) - (b + \mu) < 0$$

. This gives,

$$\frac{C_1\beta_1B + (1-\theta)b\mu N}{\mu N \left[\left(\varepsilon + \alpha + \mu - m_1\right) + \left(\eta + \mu - m_2\right) + \left(b + \mu\right) \right]} < 1 \text{ and}$$

$$\det(J) = \left[\frac{C_1\beta_1B}{\mu N} - A\right] \left[A_2(b+\mu) - \eta A_1\right] - \frac{C_2\beta_2B}{\mu N} \left(-\alpha(b+\mu) - A_1\varepsilon\right) + \left(\frac{C_3\beta_3B}{\mu N} + b\theta r_2\right) \left[\alpha\eta + A_2\varepsilon\right] > 0$$

$$=\frac{C_{1}\beta_{1}B(\mu A_{2}-A_{1}\eta)\theta br_{2}-C_{2}\beta_{2}B(-\varepsilon A_{1}-\alpha(b+\mu))+C_{3}\beta_{3}B(\alpha\eta+A_{2}\varepsilon)}{\mu N(AA_{2}\mu-AA_{1}\eta-\theta br_{2}(\alpha\eta+\varepsilon A_{2}))}=R_{0P}<1$$

2.2.2.1.2 Global stability

The corruption-free equilibrium is globally stable if $\det \left(I - \left(FV^{-1} \right) \right) > 0$.

$$\Rightarrow 1 - \frac{C_1 \beta_1 B(\mu A_2 + A_1 \eta) \theta b r_2 - C_2 \beta_2 B(\varepsilon A_1 - \alpha (b + \mu)) + C_3 \beta_3 B(\alpha \eta + A_2 \varepsilon)}{\mu N(A A_2 \mu + A A_1 \eta - \theta b r_2 (\alpha \eta + \varepsilon A_2))} > 0$$

means $R_{0P} < 1$.

2.2.3 SEIP-Model for Corruption-Free Society

The non-linear corruption model with incidence function is below:

$$\frac{dS}{dt} = B + \theta b \left(E + r_1 P \right) - \left(\lambda + \mu \right) S$$

$$\frac{dE}{dt} = \lambda S + b \theta r_2 P + (1 - \theta) b E + m_1 E - \left(\varepsilon + \alpha + \mu \right) E$$

$$\frac{dI}{dt} = \alpha E + b \left(1 - \theta r_1 - \theta r_2 \right) P + m_2 I - \left(\eta + \mu \right) I$$

$$\frac{dP}{dt} = \varepsilon E + \eta I - \left(b + \mu \right) P$$

$$C \beta S E + C \beta S I + C \beta S P$$
(6)

where,
$$\lambda = \frac{c_1 \rho_1 5L + c_2 \rho_2 5I + c_3 \rho_3 5I}{N}$$

The susceptible class is not taking in above two models. So, we analyzed model (6). Follow the same approach as discussed in section 2.2.1, we have

$$X' = \frac{dX}{dt} = \Im(X) - \upsilon(X)$$

where, $\Im(X) = \begin{bmatrix} c_1\beta_1SE + c_2\beta_2SI + c_3\beta_3SP \\ 0 \\ 0 \end{bmatrix}$ and
$$\upsilon(X) = \begin{bmatrix} (\alpha + \mu + \varepsilon - m_1)E - \theta br_2P - (1-\theta)bE \\ (\eta + \mu - m_2)I - \alpha E - b(1 - \theta r_1 - \theta r_2)P \\ -\varepsilon E - \eta I + (b + \mu)P \\ -B - \theta b(E + r_1P) - c_1\beta_1SE + c_2\beta_2SI + c_3\beta_3SP - \mu S \end{bmatrix}$$

As in section 2.2.1, the matrices F and V are calculating at corruption-free equilibrium X_0 . So, we take E = I = P = 0.

Now, the basic reproduction number of the system (6) with incidence function is the spectral radius of FV^{-1} . So,

$$R_{0} = \frac{C_{1}\beta_{1}BA_{2}\theta br_{2} - C_{2}\beta_{2}B(AA_{1} - \alpha\theta br_{2}) + C_{3}\beta_{3}BAA_{2}}{\mu N(AA_{2}\mu + AA_{1}\eta - \theta br_{2}(\alpha\eta + \varepsilon A_{2}))} = 0.35968 \quad (7)$$

The basic reproduction number $R_0 < 1$, shows that the society is corruption free.

3. Sensitivity Analysis

The sensitivity indices of all the different parameters reflect the effect of different parameter in corruption spread. It helps us for making the corruption system endemic.

The different parameters values are used in model are in Table 1. The sensitivity indices calculated on the parameters on which the value of basic reproduction number depends. The results are given in Table2.

Parameter	Sign	Value
N	-	0.0009
В	+	0.0004
μ	-	18.2099
C_1	+	0.0010
<i>C</i> ₃	+	0.0100
β_1	+	0.0060
β_3	+	2.2574

Table2.Sensitivity indices of parameters

<i>r</i> ₁	+	0.1466
<i>r</i> ₂	+	0.0559
α	+	0.0591
b	+	0.3345
θ	+	0.0576
<i>m</i> ₁	+	1.2192
<i>m</i> ₂	+	0.0397
η	-	0.0057
ε	-	0.0275

This table 2 shows that the most effective parameters in the corruption transmission are the contact rate of susceptible with punished people and migrated individuals in exposed class. The corruption free individuals make system in control.

4. Numerical Simulation

The results of simulation are useful to understand the dynamical behaviour of the system of corruption and help us to take proper decision for making corruption free future. The outcomes in different compartments are shown in following Figure 2.



Figure2. Movement of less corrupted individual to Susceptible

In Figure2, we show that the less corrupted individuals are initially not spreading corruption more means they make system stable and then after they converge to stability which makes society corruption-free.



Figure3.Transmission of Most corrupted to Susceptible

In Figure3, we clearly observe that the individuals are most corrupted in one region but after some time they becomes stable and then they turns in to non-corrupted region.



Figure 4. Punishment to the less corrupted individuals

Figure4 gives the clear idea of punishment. Here the graph goes horizontally means we give less punishment to less corrupted individuals. If we give

punishment in the beginning of the period then system goes under control immediately.



Figure 5. Punishment to most corrupted individuals

In Figure5, we observe that most corrupted people go in maximum punishment direction. We give more punishment to these infected people because arrow goes vertically here. After some time corrupted people slowly decreases this corruption habit and system turned in stable condition.

5. Conclusion

In this study, we presented and analysed the non-linear mathematical model for corruption using incidence function. Job transfers and suspension are considered here as a punishment for individuals. The basic reproduction number for migration and *EI*-model, model for punishment to individual and the total mass incidence *SEIP*-model are worked out. The corruption–free equilibrium model is locally asymptotically stable whenever $R_0 < 1$ and the model is global stability when det $(I - FV^{-1}) > 0$. In the upcoming year, this model may be helpful to society to reduce the burden of corruption. Punishment to the individual is must for all irrespective of the position they held. Some people believed that by changing some law or revitalizing some old law with new rotations, we can moderate the stronghold of corruption. We can educate people and developed good values in them, so they may not be corrupted.

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